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$$z=0 \rightarrow z^2 = x^2 + y^2$$

$$z=2 \rightarrow 2^2 = x^2 + y^2 - 7 \rightarrow (4+7) = x^2 + y^2$$

$$f = \begin{bmatrix} x_1 - x_3 \\ x_1 + x_2 \\ x_3 + 1 \end{bmatrix} \rightarrow \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix}$$

3 flächen: $\text{rad (Krum)}, \text{toppsirke}, \text{bunnsirke}$
 $((\text{Krum}), \text{topp}, \text{bunn})$

$$\underbrace{\iint_{\text{Krum}} f \cdot \hat{n} \, ds}_{\text{0lem}} = \underbrace{\iint_{\partial T} f \cdot \hat{n} \, ds}_{\substack{\uparrow \\ \text{Divergenztheorem}}} - \underbrace{\iint_{\text{bunn}} f \cdot \hat{n} \, ds}_{\text{0mille}} - \iint_{\text{topp}} f \cdot \hat{n} \, ds$$

$$= \underbrace{\iiint_T \nabla \cdot f \, dV}_{(1)} - \underbrace{\iint_{\text{bunn}} f \cdot \hat{n} \, ds}_{(2)} - \underbrace{\iint_{\text{topp}} f \cdot \hat{n} \, ds}_{(3)}$$

$$(1) \quad \nabla \cdot f = 1 + 1 + 1 = 3$$

$$3 \cdot \underbrace{\iiint_T dV}$$

$$T = \left\{ (s, \theta, z) : \begin{array}{l} 0 \leq s \leq \sqrt{z^2 + 7} \\ 0 \leq \theta \leq 2\pi \\ 0 \leq z \leq 2 \end{array} \right.$$

Sylinderkoordinaten.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \cos \theta \\ s \sin \theta \\ z \end{bmatrix} \rightarrow \begin{bmatrix} s \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{x_1^2 + x_2^2} \\ \tan^{-1}\left(\frac{x_2}{x_1}\right) \\ x_3 \end{bmatrix} \leftrightarrow s^2 = x_1^2 + x_2^2$$

$$z = \sqrt{s^2 - 7}$$

$$z^2 = s^2 - 7$$

$$s^2 = z^2 + 7$$

$$s = \sqrt{z^2 + 7}$$

NB!

$$|J| = 5$$

$$\begin{aligned} 3 \cdot \int_0^{2\pi} \int_0^{\sqrt{z^2+7}} \int_0^{\sqrt{z^2+7}} s \, ds \, d\theta \, dz &= 3 \cdot \int_0^{2\pi} \int_0^{\sqrt{z^2+7}} \left[\frac{1}{2} s^2 \right]_0^{\sqrt{z^2+7}} d\theta \, dz \\ &= 3 \cdot \frac{50\pi}{3} \\ &= \underline{\underline{50\pi}} \end{aligned}$$

$$\textcircled{2} \quad \iint_{\text{bunn}} f \cdot \hat{n} \, ds = \iint_{\text{bunn}} -f_3 \, ds = \iint_{\text{bunn}} \underbrace{f(x_1, x_2, 0)}_{[x_1, x_2, 1]} \cdot (0, 0, -1) \, ds$$

$$\begin{aligned} \text{bunn} &= \left\{ (r, \theta) : \begin{array}{l} 0 \leq r \leq \sqrt{7} \\ 0 \leq \theta \leq 2\pi \end{array} \right\} \\ &= -1 \cdot \iint_{\text{bunn}} ds \quad \begin{array}{l} \nearrow \text{Areal der bunnflächen} \\ \searrow \text{Areal der s\u00fcckel} \end{array} \\ &= -1 \cdot \int_0^{2\pi} \int_0^{\sqrt{7}} r \, dr \, d\theta \\ &= -1 \cdot \pi r^2 = -1 \cdot 7\pi \\ &= \underline{\underline{-7\pi}} \end{aligned}$$

③

$$\iint_{\text{topp}} f \cdot \hat{n} \, ds = \iint [x_1 - 2, x_1 + x_2, 3] \cdot [0, 0, 1] \, ds$$

$\uparrow$
 $f(x_1, x_2, 2)$

$$= 3 \iint_{\text{topp}} ds = 3 \cdot \pi r^2$$

$$= 3 \cdot \pi \cdot 11$$

$$= 33\pi$$

Endelig svar

$$\iint_{\text{Krum}} f \cdot \hat{n} \, ds = \textcircled{1} - \textcircled{2} - \textcircled{3}$$

Krum

$$= 50\pi - (-7\pi) - 33\pi$$

$$= \underline{\underline{24\pi}}$$