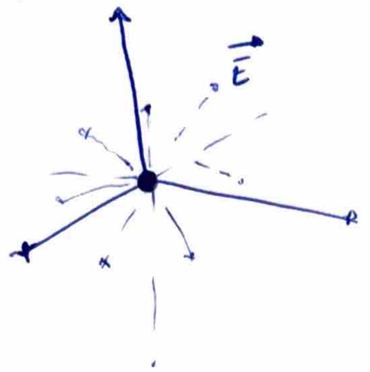
4

$\vec{F} = q\vec{E}$

↑ Den opplevde $\frac{1}{q}$ av kraften en positiv partikkel m/ ladning q ville kjent.



Sfærisk symmetri i $\vec{E}$

↑ lik mtp noe.

Translasjonssymmetri → Δ → Δ — Δ

Forflytning uten rotasjon.

Rotasjonssymmetri → endrer seg ikke ved rotasjon

Sfærisk symmetri → alle punkter på ett og samme sfære er like

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \to \iiint_{\Omega} \nabla \cdot \vec{E}\, dV = \frac{1}{\varepsilon_0} \overbrace{\iiint_{\Omega} \rho\, dV}^{\text{Hva er dette?}}$$

Total ladning inne i Ω.

Hva burde denne være? Kuleskall i origo

Div.teo.

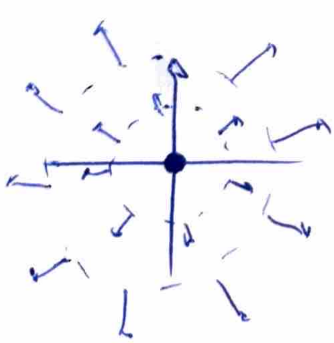
$$\oiint_{\partial\Omega} \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} Q$$

Konstant på kuleskallet

Parallelle

ladning til p.ladn.

Alle punkter her samme r



Radielt felt.

$\vec{E}(r)$

$$\vec{E}(r) \underbrace{\oiint_{\partial\Omega} dS}_{\text{Areal av kuleskall}} = \frac{Q}{\varepsilon_0}$$

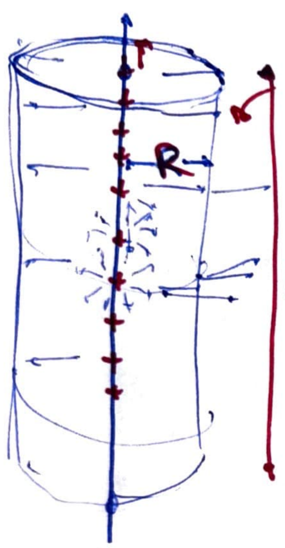
$= 4\pi r^2$

$$\vec{E}(r) = \frac{Q}{\varepsilon_0 4\pi r^2}\hat{r} \propto \frac{1}{r^2}$$

5 Påstand. $\iint_{\partial\Omega} B\cdot\hat{n}\cdot ds = 0$, vis dette.

Hint. div teorem

6 Vi gjør denne med elektrisk feltstyrke rundt ledning isteden $\rightarrow$ {fucked ting vi må akseptere, strøm er definert som forflytning av positive hull ...}



Symmetriargument. $\hat{E}(r)$

$$\begin{bmatrix}\hat{\ }\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \begin{bmatrix}0\\0\\-1\end{bmatrix}$$

$$\oiint_{\text{gaussflate}} \vec{E}\cdot\hat{n}\,ds = \iint_{\text{Sidevegger}} \underbrace{\vec{E}\cdot\hat{n}}_{\text{Paralell}} ds + \underbrace{\iint_{\text{topp Plate}} \vec{E}\cdot\hat{n}\,ds + \iint_{\text{bunn Plate}} \vec{E}\cdot\hat{n}\,ds}_{=0}$$

Konstant r.

$$= \vec{E}(r) \iint_{\text{Sidevegger}} ds$$

$$= \vec{E}(r)\,\underbrace{2\pi R\cdot L}_{\text{g. lengde}}$$

høyde

Gauss law: $\oiint \vec{E}\cdot\hat{n}\,ds = \frac{1}{\varepsilon_0} \iiint \rho\, dV$ — total ladning.

$$= \frac{1}{\varepsilon_0}\int \rho_L\, dl$$

Hvis konstant

$$= \frac{1}{\varepsilon_0}\rho_L\cdot L$$

$$\vec{E}(r) = \frac{\rho_L}{\varepsilon_0\, 2\pi R}$$

Fra Power Point.

$\nabla \cdot \nabla \phi$

Slide 1.) Poisson ligning: $\Delta \phi = f$ / $\nabla^2 \phi = f$.

↳ laplace operator.

Statisk tilfelle = alt er statisk / ingen endring i tid

$\dot{E} = 0$ $\dot{B} = 0$

$$\left.\begin{array}{ll} \nabla \cdot E = \frac{\rho}{\varepsilon_0} & \nabla \cdot B = 0 \\ \nabla \times E = 0 & \nabla \times B = \frac{J}{\varepsilon_0} \end{array}\right\} \text{statisk Dekoblet}$$

Curl lik 0 → konservativt

↳ $E = -\nabla \phi \xrightarrow{\text{gauss}} \nabla \cdot E = \nabla \cdot (-\nabla \phi) = \frac{\rho}{\varepsilon_0}$

$-\Delta \phi = \frac{\rho}{\varepsilon_0}$

Er dette Poisson? Ja → $\boxed{\Delta \phi = -\frac{\rho}{\varepsilon_0}}$

→ Dersom tomt rom $(\rho = 0)$

$\Delta \phi = 0$ laplaces ligning !!

→ Husk av resten.

Slide 1) Hva er bølge ligningen?

$$\frac{\partial^2 u}{\partial t^2} = c^2 (\Delta u)$$ } Bcom Wikipedia.

Tomt rom → ingen partikler, ingen ladning, ingen ladning i bevegelse ↗ strøm.

$\rho = 0$ $\quad J = 0$

$\nabla \cdot E = 0$ $\quad \nabla \cdot B = 0$

(2) $\nabla \times E = -\frac{\partial B}{\partial t}$ $\quad$ (1) $c^2 \nabla \times B = \frac{\partial E}{\partial t}$

} Tomt rom.

obs vakum impresser ikke tomt rom.

(1) ~~[illegible]~~ Deriverer mhp tid.

$$c^2 \nabla \times \frac{\partial B}{\partial t} = \frac{\partial^2 E}{\partial t}$$

$$c^2 \nabla \times (-\nabla \times E) = \frac{\partial^2 E}{\partial t}$$

$$-c^2 (\nabla(\underbrace{\nabla \cdot E}_{0}) - \Delta E) = \frac{\partial^2 E}{\partial t}$$

$$\boxed{c^2 \Delta E = \frac{\partial^2 E}{\partial t}} \to \text{Bølge}$$

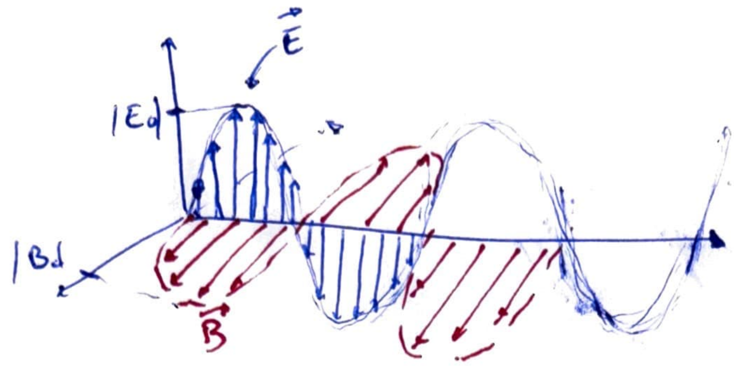
(2)

$$\nabla \times \left(\frac{\partial E}{\partial t}\right) = -\frac{\partial^2 B}{\partial t}$$

⋮

Bølgeligning for magnet felt.

Slide 2)



vektor fordi $|E_0|\hat{e}_E = E_0$

$$\vec{E}(\vec{x},t) = \vec{E}_0 \, \underbrace{e^{i(\vec{k}\cdot\vec{x} - ct)}}_{\text{kompleks tall}}$$

$$\vec{B}(\vec{x},t) = \underbrace{\vec{B}_0}_{\text{vektor}} \, e^{i(\vec{k}\cdot\vec{x} - ct)}$$

vektor i propagasjonsretning.

OBS! $e^{ix} = \cos(x) + i\sin(x)$

$$e^{i(\vec{k}\cdot\vec{x} - ct)} = \cos(\underbrace{\vec{k}\cdot\vec{x}}_{\text{skalar}} - ct) + i\sin(\vec{k}\cdot\vec{x} - ct)$$

Vi skal vise at $\vec{E}_0 \perp \vec{B}_0 \perp \vec{k}$

(1) $\vec{k} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\vec{k}\cdot\vec{x} = (k_1 x_1 + k_2 x_2 + k_3 x_3)$

(2) $\vec{E}_0 = \begin{bmatrix} E_{01} \\ E_{02} \\ E_{03} \end{bmatrix}$ $\vec{E} = \begin{bmatrix} E_{01} \\ E_{02} \\ E_{03} \end{bmatrix} e^{i(\vec{k}\cdot\vec{x} - ct)}$

$E_1 = E_{01} e^{i(kx - ct)}$

$E_2 = E_{02} e \ldots$

$E_3 = E_{03} e \ldots$

$$\nabla \times E = \begin{bmatrix} \frac{\partial E_3}{\partial x_2} - \frac{\partial E_2}{\partial x_3} \\ \frac{\partial E_1}{\partial x_3} - \frac{\partial E_3}{\partial x_1} \\ \frac{\partial E_2}{\partial x_1} - \frac{\partial E_1}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial E_3}{\partial x_2} = \vec{E}_{03}\, e^{i(\vec{K}\cdot\vec{x} - ct)}\, \vec{K}_2\, i$$

$$\frac{\partial E_3}{\partial x_1} = \vec{E}_{03}\, e^{i(\vec{K}\cdot\vec{x} - ct)}\, \vec{K}_1\, i$$

$$\frac{\partial E_2}{\partial x_3} = \vec{E}_{02}\, e^{i(\vec{K}\cdot\vec{x} - ct)}\, \vec{K}_3\, i$$

etc. . . .

$$\underline{\nabla \times E} = \begin{bmatrix} E_{03} \underline{K_2} - E_{02} \underline{K_3} \\ E_{01} \underline{K_3} - E_{03} \underline{K_1} \\ E_{02} \underline{K_1} - E_{01} \underline{K_2} \end{bmatrix} \underbrace{i\, e^{i(\vec{K}\cdot\vec{x} - ct)}}_{\text{tall (Komp. but still)}}$$

$$= (\vec{K} \times \vec{E}_0)\, i\, e^{i(\vec{K}\cdot\vec{x} - ct)}$$

$$\underline{\nabla \times E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left(\vec{B}_0\, e^{i(\vec{K}\cdot\vec{x} - ct)}\right)$$

$$= -\vec{B}_0\, e^{i(\vec{K}\cdot\vec{x} - ct)}\, (-ic)$$

2 uttrykk for $\nabla \times E$

$$(\vec{K} \times \vec{E}_0)\, i\, e^{i(\vec{K}\cdot\vec{x} - ct)} = \div\, \vec{B}_0\, e^{i(\vec{K}\cdot\vec{x} - ct)}\, (\div\, i\, c)$$

$$\vec{K} \times \vec{E}_0 = c\, \vec{B}_0$$

~~Kompleks~~ skalar

Dette viser at : $\vec{K} \perp \vec{B}_0$

$\vec{E}_0 \perp \vec{B}_0$

Mangler ! $\vec{K} \perp \vec{E}_0$

↳ Gjør akkurat samme m/ $c^2 \nabla \times B$

Slide 4) Vi skal komme fram til:

$$\dot{\rho} + \nabla \cdot \vec{J} = 0$$

Hvilken lover har disse symbolene?

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

Har lyst på denne inn

$$c^2 \nabla \times B = \frac{J}{\varepsilon_0} + \frac{\partial E}{\partial t}$$

Denne har på en måte begge

$$\nabla \cdot (c^2 \nabla \times B) = \nabla \cdot \frac{J}{\varepsilon_0} + \frac{\partial}{\partial t}(\nabla \cdot E) \quad \Big| \nabla \cdot$$

På begge sider

$$c^2 \underbrace{(\nabla \cdot (\nabla \times B))} = \frac{1}{\varepsilon_0} \nabla \cdot J + \frac{\partial}{\partial t} \frac{\rho}{\varepsilon_0} \quad \Big| \cdot \varepsilon_0$$

Divergensen av curl er alltid 0

$$0 = \nabla \cdot J + \dot{\rho}$$

Boom!

Mer fysikk.

Total ladning i et volum Q er gitt ved:

$$Q = \iiint_{\Omega} \rho \, dV$$

en hver endring i Q må skyldes at ladning flytter seg ut / inn gjennom randen til Ω, $\partial \Omega$

Altså det er en strøm fluks over randen.

$$\oiint_{\partial \Omega} \vec{J} \cdot \hat{n} \, dS = -\frac{\partial}{\partial t} \iiint_{\Omega} \rho \, dV$$

$$\iiint_{\Omega} \nabla \cdot J \, dV = \iiint -\frac{\partial \rho}{\partial t} dV \rightarrow \nabla \cdot J = -\dot{\rho}$$

Slide 5) Finn A slik at

① $\nabla \times \vec{A} = \vec{f}$ og ② $\nabla \cdot \vec{A} = 0$

$$f = \begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix} \qquad A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \qquad \nabla \times A = \begin{bmatrix} \frac{\partial A_3}{x_2} - \frac{\partial A_2}{x_3} \\ \frac{\partial A_1}{x_3} - \frac{\partial A_3}{x_1} \\ \frac{\partial A_2}{x_1} - \frac{\partial A_1}{x_2} \end{bmatrix}$$

$$\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3} = x_3 \qquad A_2(x_3) = -\frac{1}{2}x_3^2$$

$$\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1} = x_1 \qquad A_3(x_1) = -\frac{1}{2}x_1^2$$

$$\frac{\partial A_2}{\partial x_1} \quad \frac{\partial A_1}{\partial x_2} = x_2 \qquad A_1(x_2) = -\frac{1}{2}x_2^2$$

① $$\vec{A} = \begin{bmatrix} x_3^2 \\ x_1^2 \\ x_2^2 \end{bmatrix}\left(-\frac{1}{2}\right) \begin{matrix} \leftarrow A_1 \\ \leftarrow A_2 \\ \leftarrow A_3 \end{matrix}$$

Sjekker krav ②: Er $\nabla \cdot \vec{A} = 0$

$$\nabla \cdot \vec{A} = \left\{ \frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3} = 0 \right.$$

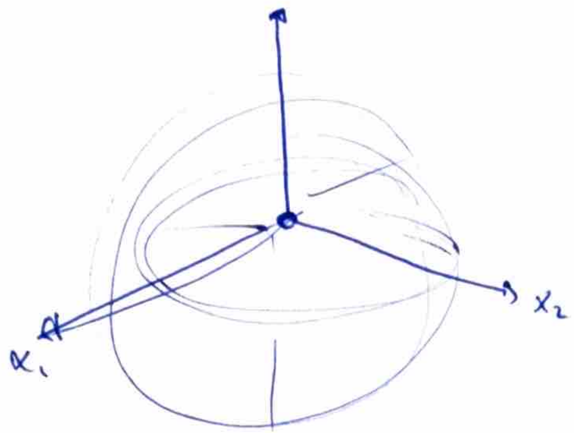
✓

Er f konservativt? NEI!

$\boxed{\nabla \times f = 0 \Leftrightarrow \text{konservativ}}$

Men $\nabla \times f \neq 0$

Slide 6)



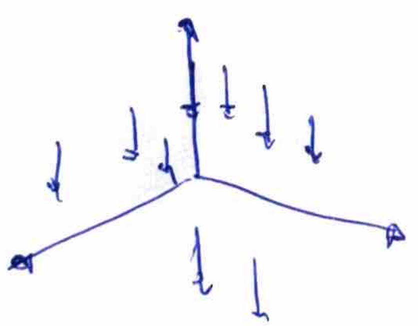
$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\hookrightarrow \iiint_\Omega \nabla \cdot E \, dV = \iiint_\Omega \frac{\rho}{\varepsilon_0} \, dV$$

$$\underbrace{\oiint_{\partial\Omega} \vec{E} \cdot \hat{n} \, dS}_{\text{total fluks}} = \frac{q}{\varepsilon_0}$$

total fluks → svaret

Slide 7.)



$$\vec{F} = m\vec{g}$$

$$\vec{g} = \begin{bmatrix} 0 \\ 0 \\ -9.81\,m/s^2 \end{bmatrix}$$

$$\vec{F} = \begin{bmatrix} 0 \\ 0 \\ -m \cdot 9.81\,m/s^2 \end{bmatrix}$$

Skalar felt

$$\vec{F} = -\underbrace{\nabla \phi}$$

$$= -\left[\frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} + \underbrace{\frac{\partial \phi_3}{\partial x_3}}\right] \rightarrow \phi_3 = m|g|\,x_3 + C$$

ϕ_1, ϕ_2

$$\frac{\partial \phi_3}{\partial x_3} = m \cdot |g| \rightarrow \phi_3 = m \cdot |g|\,x_3 + C$$

$$\phi(x) = m|g|\,x_3 \boxed{+\,C}$$